CIE3109
Structural Mechanics 4

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Module : Unsymmetrical and/or inhomogeneous cross section

LECTURE 4

v2021

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CIE3109 : Structural Mechanics 4

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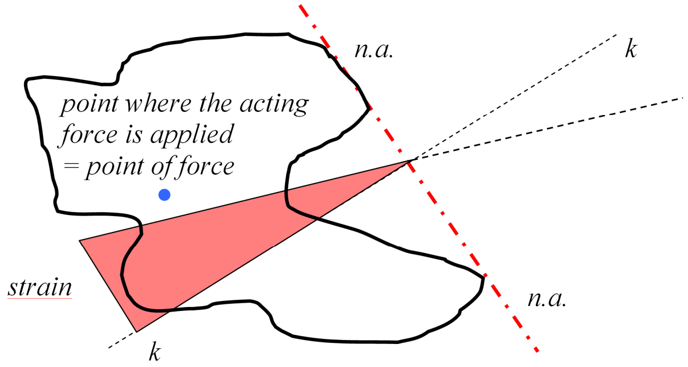
CORE

If all stresses within the cross section have the same sign (either compression or tension) due to an applied force this force has to be applied within a bounded area which is called the core.



Strategy

Find all possible positions of neutral axis which are just outside of the cross section. Each position of a valid neutral axis will give a point which is a point of the core boundary.

What do we need:

1) Relate the position of a neutral axis to the coordinate system used
2) Relate the neutral axis to the point of application of a load at the core boundary

Position of the neutral axis

strain:

$$\varepsilon(y,z) = \varepsilon + \kappa_y y + \kappa_z z$$

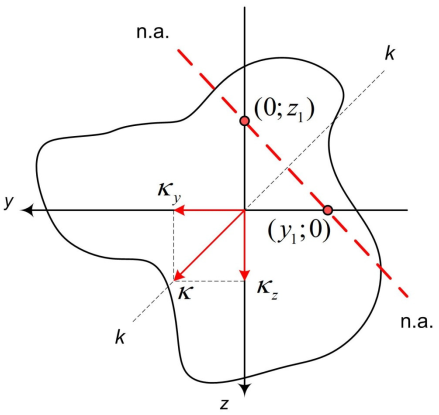
neutral axis:

$$\varepsilon + \kappa_y y + \kappa_z z = 0 \quad \Leftrightarrow$$

$$1 + \frac{\kappa_y}{\varepsilon} y + \frac{\kappa_z}{\varepsilon} z = 0$$

through $(y_1, 0)$ *and* $(0, z_1)$:

$$y_1 = -\frac{\varepsilon}{\kappa_y}; \quad z_1 = -\frac{\varepsilon}{\kappa_z};$$



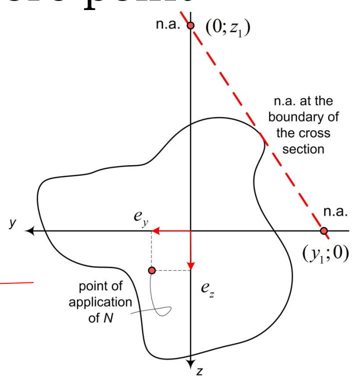
Relate neutral axis to core point

$$\begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EA & & \\ & EI_{yy} & EI_{yz} \\ & EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa_y \\ \kappa_z \end{bmatrix} \quad (1)$$

with: $N \neq 0$

and:

$$\begin{bmatrix} M_y \\ M_z \end{bmatrix} = N \begin{bmatrix} e_y \\ e_z \end{bmatrix} = EA\varepsilon \begin{bmatrix} e_y \\ e_z \end{bmatrix}$$



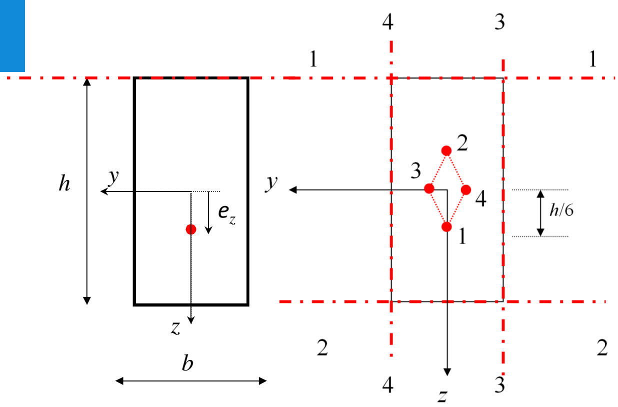
substitute in (1):

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} \kappa_y / \varepsilon \\ \kappa_z / \varepsilon \end{bmatrix}$$

use result from previous slide:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

EXAMPLE 7



e.g. line **1-1**

n.a. through points :

(*b*/2;-*h*/2) and (-*b*/2; -*h*/2)

Thus:

y1=∞

z1=-*h*/2

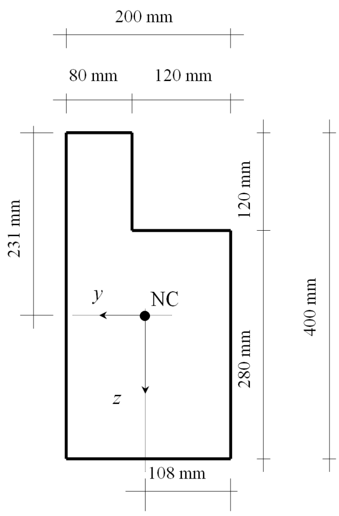
elaborate:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{bh}\begin{bmatrix} \frac{1}{12}hb^3 & 0 \\ 0 & \frac{1}{12}bh^3 \end{bmatrix}\begin{bmatrix} 0 \\ \frac{2}{h} \end{bmatrix}$$

$$e_y = 0; \qquad e_z = \frac{h}{6}$$

For each side (position of the *n.a.*) a point of force can be determined, this is a corner of the core.

EXAMPLE 8



Step 1

NC towards the top:

$$z_{NC} = \frac{400\times200\times200-120^2\times60}{400\times200-120^2} = 231\text{ mm}$$

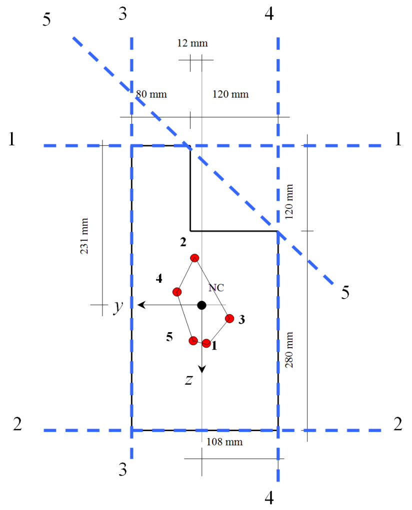
NC towards the right:

$$y_{NC} = \frac{400\times200\times100-120^2\times60}{400\times200-120^2} = 108\text{ mm}$$

Step 2 : "Two Letter" symbols:

$$EA = E\times65600 \quad \text{N}$$
$$EI_{yy} = E\times231.3\times10^6 \text{ Nmm}^2$$
$$EI_{yz} = -E\times98.3\times10^6 \text{ Nmm}^2$$
$$EI_{zz} = E\times705.2\times10^6 \text{ Nmm}^2$$

Result

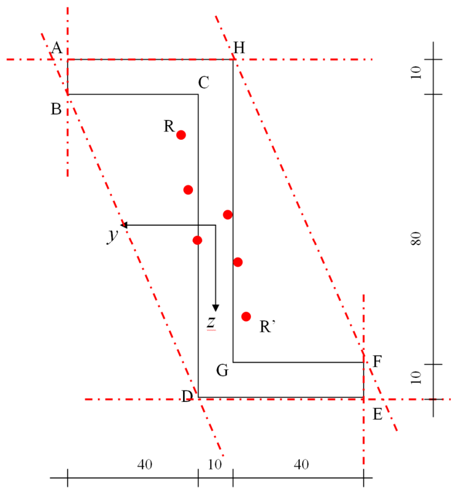


STEP 3 : Solve for each side:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = -\frac{1}{EA}\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{zy} & EI_{zz} \end{bmatrix}\begin{bmatrix} 1/y_1 \\ 1/z_1 \end{bmatrix}$$

line	y_1	z_1	*core point*	e_y	e_z
1-1	∞	-231	1	-6.5	46.5
2-2	∞	169	2	8.9	-63.6
3-3	92	∞	3	-36.6	16.3
4-4	-108	∞	4	31.2	-13.9
5-5	-219	-219	5	8.6	42.2

Assignment



$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = -\frac{1}{EA}\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{zy} & EI_{zz} \end{bmatrix}\begin{bmatrix} 1/y_1 \\ 1/z_1 \end{bmatrix}$$

line	y_1	z_1	e_y	e_z	*point*
A-B	+45	∞	-7.59	11.11	
B-D	27.22	61.25	-4.39	-3.94	
D-E	∞	50	+10	-27.33	*R*
E-F	-45	∞	7.59	-11.11	
F-H	-27.22	-61.25	4.39	3.94	
A-H	∞	-50	-10	27.33	*R'*